MILLlKAN'S OIL DROP EXPERIMENT

Measurement of the Charge of the Electron by the Millikan Method

Introduction

Robert Andrews Millikan was an important person in the development of physics. Best known for his oil drop experiment, Robert Millikan also verified experimentally the Einstein equation for the photoelectric effect. For these two investigations, he was awarded the Nobel Prize in 1923.
The charge of the electron was determined by Millikan after an exhausting research effort measuring the charge on oil droplets. Oil droplets have the advantage over other liquid droplets that they almost do not evaporate, and hence have a (quasi) stable mass. A schematic view of the Millikan apparatus is shown in Figure 1.

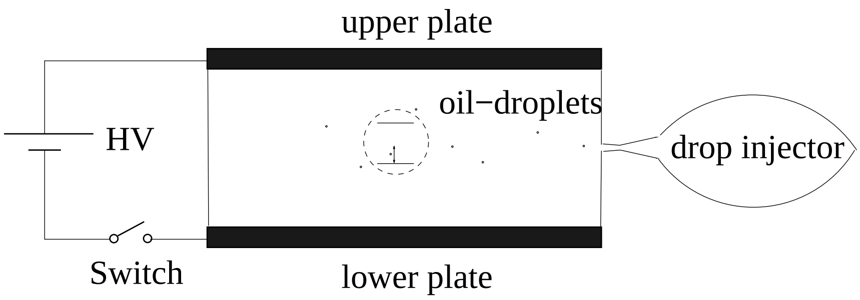

Figure 1: A schematic view of the Millikan oil-drop apparatus. The region where a droplet can be seen by the microscope/video-camera is sketched (circle).

Oil droplets charged by an atomizer are allowed to pass through a small hole between the plates of a parallel-plate capacitor. If these droplets are illuminated from the side, they appear as brilliant stars against a dark background, and the rate of fall of individual drops may be determined. While Millikan observed the droplets using a microscope, in our lab an attached video camera will aid the visualization of the droplets and the determination of their velocity. A schematic view of the apparatus setup is shown in Figure 1.
If an electrostatic field of several thousand volts per meter is applied to the capacitor plates, the drop may move slowly upward, typically at rates of *tenths* of a millimeter per second. Because the rate of fall is comparable, a single droplet with constant mass and radius may be followed for hours, alternately rising and falling, by simply turning the electric field on and off.
In the original setup Millikan used an ionizer (source of ionizing radiation) to change the *amount* of charge on the droplets. With changing charge, the velocity of the droplets changes if the electric field is present (rise/up), while it remains the same if not (fall/down). Measuring these *jumps* of velocity gives an ideal database to determine the electric charge. However, this procedure is difficult and cumbersome, for that we follow a slightly different approach.

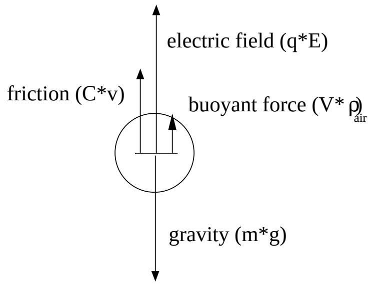

Figure 2: The forces on a charged oil droplet in the Millikan experiment.

We will measure the fall/rise velocities of different droplets and, by doing this, we will determine a charge for *each droplet*, which for each droplet will be a integer number times the electron charge.

The quantitative analysis of the Millikan experiment starts with Newton's second law applied to the oil drop, that means the total force is the sum of all forces working on the droplet ($\sum F_y = ma_y$, see Fig. 2). We assume that the oil-drops are composed of spherical droplets having a constant mass. Assuming this, we can write the single forces as the gravitational force F_g

$$F_g = mg = \frac{4\pi}{3}\rho r^3 g \tag{1}$$

the electrical force F_q

$$F_q = qE \tag{2}$$

and the friction force D given by Stokes' law

$$D \;=\; 6\pi r\eta v \tag{3}$$

Here m is the droplet mass, g the standard acceleration of gravity, q the droplet charge, E the electric potential, r is the droplet radius, η is the viscosity of air and v is the speed of the droplet. Because the friction force, D, is large, a constant velocity of fall/rise is quickly achieved. This means the drops are not accelerated, thus $a_y = 0$, or $\Sigma F_y = 0$. This leads to the expression

$$F_y \;= F_g - D \qquad = \frac{4\pi}{3}\rho r^3 g - \;6\pi r\eta v_0 \qquad = 0 \quad (\text{field off}) \tag{4}$$

$$F_y \;= F_g - D - F_e = \frac{4\pi}{3}\rho r^3 g - \;6\pi r\eta v_+ - qE = 0 \quad (\text{field on}) \tag{5}$$

Subtracting these two equations gives

$$q \;=\; \frac{6\pi r\eta\,(v_0 - v_+)}{E} \tag{6}$$

where v_+ and v_0 are the drop velocities with and without the electric field, respectively.

If we now divide the charge of different droplets q_1 and q_2, the result should be a fraction which can be reproduced by a fraction of (small) integer numbers, since

$$\frac{q_1}{q_2} = \frac{n_1 \cdot e}{n_2 \cdot e} = \frac{n_1}{n_2} \tag{7}$$

If this is the case Equation 7 constitutes a remarkably direct and powerful proof of the discreteness of charge. The value for the elementary charge can then by found by rewriting Equation 7

$$e_1 = \frac{q_1}{n_1} = \frac{q_2}{n_2} = e_2 \tag{8}$$

Up to this point our arguments have been quite general and have assumed only that the friction force on the droplet is proportional to its velocity. To determine the actual value of the electronic charge, e, the radius of the drop must be determined, as can be seen from Equation 6. The droplet radius r may be determined from Equation 5

$$r = \sqrt{\frac{9\eta v}{2\rho g}} \quad . \tag{9}$$

where η is the viscosity of air and v is the terminal speed of the droplet.
This value of r, in turn, can be used to find m from the oil density, ρ. Now the mass of the droplet can be expressed by density ρ and volume V as

$$m = \rho \cdot V = \rho \frac{4}{3}\pi a^3 \quad . \tag{10}$$

Additional corrections

Stokes' law is only approximately correct for tiny spheres moving through a gas. It holds quite accurately for a 0.1 cm radius droplet moving through a gas or for any case where the moving object radius, r, is large compared with the mean free path, L, of the surrounding molecules. In the Millikan experiment, however, r is of the same order of magnitude as the mean free path of air at STP. Consequently Stokes' law overestimates the friction force, because the droplet actually moves for appreciable times through a frictionless "vacuum". Millikan corrected Stokes' law by using a friction force which magnitude is

$$D = \frac{6\pi a\eta v}{1 + \alpha(L/a)} \tag{11}$$

and found that $\alpha = 0.81$ gave the most consistent values of e for drops of different radii. Further corrections to Stokes' law were made by Perrin and Roux, and corrections to Stokes' law and the correct value of e remained a controversial issue for more than 20 years.

In addition, Equation 1 is not correct, since we have to consider also the buoyant force of the air surrounding the droplets. This modifies Equation 1 to

$$F_g = mg - V\rho_{air}g = \frac{4\pi}{3}\rho_{oil}r^3 g - \frac{4\pi}{3}\rho_{air}r^3 g = \frac{4\pi}{3}\left(\rho_{oil} - \rho_{air}\right) g \tag{12}$$

where V is the volume of the droplet and $\rho_{air} = 1.292 kg/m^3$ is the density of air at $20°C$. The viscosity of air changes with the temperature. Its value is

$$[(18.43 \pm 0.05) + 0.05(T/C^{\circ} - 25)] \times 10^{-6} kg\ m^{-1} s^{-1}. \quad (13)$$

The actual temperature, T, in the lab has to be measured in order to take this effect into account.

Experimental Determination of e

The currently accepted value of the magnitude of the elementary charge is

$$e = 1.602176462(63) \times 10^{-19}\,C. \tag{14}$$

Experimental Task:

Measure for 10 different droplets 10 times the fall and rise time.

Data analysis:

1. Incorporate the corrections for Stokes' law, the buoyant force and the viscosity of air into Equation 4, 5 and 6. Are all the corrections necessary?
   Estimate a value for the mean free path, L, by assuming equidistant molecules in a given volume (e.g. $1mol$ in $22.4l$).

2. If the oil density is (0.890 ± 0.001) g/cm^3, find the radius, volume, and mass of the drops used in this experiment.

3. Calculate the mean velocities v_+ and v_0 for each droplet. Calculate the charge using Equation 6 and also using the one derived in question 1 for each drop. Is there a significant difference? If so what is the influence of each correction? Find the integer numbers n_1 and n_2 according to Equation 7. Are the quotients rational? Does this give a consistent picture? (If $\frac{q_1}{q_2} = \frac{3}{2}$ and $\frac{q_3}{q_2} = \frac{4}{2}$ then **has to be** $\frac{q_3}{q_1} = \frac{4}{3}$.) The distance of the markers used for the time measurement is (1.23 ± 0.1) mm.

4. Calculate the value of the elementary charge e using each droplet. Plot the values into a histogram. What is the shape of the histogram? Is it a Gaussian distribution? What is the variance and the FWHM (**F**ull **W**idth (at) **H**alf (of the) **M**axium)? Remember that the electrical field in a parallel plate capacitor is given by

   $$E = \frac{U}{d}$$

   with U the voltage and d the plate distance.

Questions

1. Are your measurements convincing evidence that charge is quantized, or are the errors so large that your measured charge appears continuous?

2. If the charge is seen to be quantized, use all of your data to produce a measured fundamental charge.

3. What is the difference between your measurement and 1.602×10^{-19} C? What is the expected error in your measurement due to the scatter of the measurements assuming all other quantities are accurately known? What is the systematical error? What would you suggest as an improvement for the millikan oil drop experiment.

Preparative tasks

- Further study : chapter 3 page 103 - page 109 of the text book ”Modern Physics” by Serway, Moses and Moyer, Saunders College Publishing, 1989

Reminder

- You should use a (that means **one**) bound lab book. If you use a computer, glue the printout into your book.
- The goal of this lab is to **learn** and to **think**! If you want to add additional work/discussion - please do it. If you feel a question/task is not very useful, feel free to leave it away. But you always have to argue **why** !
- Feedback is very welcome.
- To all graphs you have to draw (x and y!) error bars. Discussions of errors is also required. This does not mean that you always have to do all the calculation according to error propagation rules. If this according to your arguments is not necessary (statistical errors much smaller then systematical, one error source rules everything, ...) you do not have to do it. However, you have to discuss it.
- An answer to a question is always a whole sentence. A value (almost) always has an unit.
- And: always remember to have fun, in the lab - and also in the Neustadt and at the river Elbe.